OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/01 Pure Mathematics

Practice Paper – Set 4

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

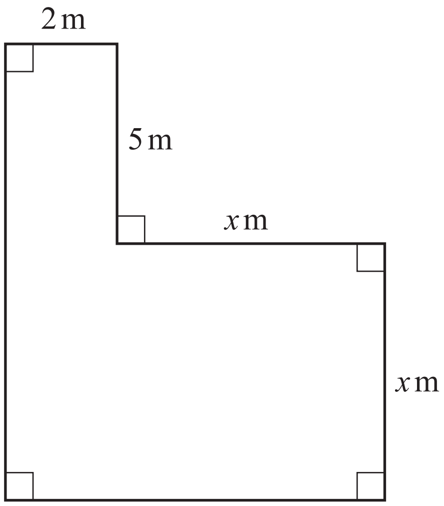
Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Andrea is comparing the prices charged by two different taxi firms.
Firm **A** charges £20 for a 5 mile journey and £30 for a 10 mile journey, and there is a linear relationship between the price and the length of the journey.
Firm **B** charges a pick-up fee of £3 and then £2.40 for each mile travelled.

Find the length of journey for which both firms would charge the same amount. **[4]**

2



The diagram shows a patio.

The perimeter of the patio has to be less than 44 m.

The area of the patio has to be at least 45 m^2.

(a) Write down, in terms of x, an inequality satisfied by

(i) the perimeter of the patio, **[1]**

(ii) the area of the patio. **[1]**

(b) Hence determine the set of possible values of x. **[4]**

3 **In this question you must show detailed reasoning.**

(a) Given that $\sin\alpha = \frac{2}{3}$, find the exact values of $\cos\alpha$. **[2]**

(b) Given that $2\tan^2\beta - 7\sec\beta + 5 = 0$, find the exact value of $\sec\beta$. **[4]**

4 **In this question you must show detailed reasoning.**

Solve the simultaneous equations

$$\mathrm{e}^x - 2\mathrm{e}^y = 3$$
$$\mathrm{e}^{2x} - 4\mathrm{e}^{2y} = 33.$$

Give your answer in an exact form. **[5]**

5 **(a)** Given that $\mathrm{f}(x) = x^2 - 4x$, use differentiation from first principles to show that $\mathrm{f}'(x) = 2x - 4$. **[5]**

(b) Find the equation of the curve through (2, 7) for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 4$. **[3]**

6 **In this question you must show detailed reasoning.**

A sequence S has terms $u_1, u_2, u_3 \ldots$ defined by $u_1 = 500$ and $u_{n+1} = 0.8u_n$.

(a) State whether S is an arithmetic sequence or a geometric sequence, giving a reason for your answer. **[1]**

(b) Find u_{20}. **[2]**

(c) Find $\sum_{n=1}^{20} u_n$. **[2]**

(d) Given that $\sum_{n=k}^{\infty} u_n = 1024$, find the value of k. **[5]**

7 As a spherical snowball melts its volume decreases. The rate of decrease of the volume of the snowball at any given time is modelled as being proportional to its volume at that time. Initially the volume of the snowball is $500\,\text{cm}^3$ and the rate of decrease of its volume is $20\,\text{cm}^3$ per hour.

(a) Find the time that this model would predict for the snowball's volume to decrease to $250\,\text{cm}^3$. **[7]**

(b) Write down one assumption made when using this model. **[1]**

(c) Comment on how realistic this model would be in the long term. **[1]**

8 **(a)** Expand $\sqrt{1+2x}$ in ascending powers of x, up to and including the term in x^3. **[4]**

(b) Hence expand $\dfrac{\sqrt{1+2x}}{1+9x^2}$ in ascending powers of x, up to and including the term in x^3. **[3]**

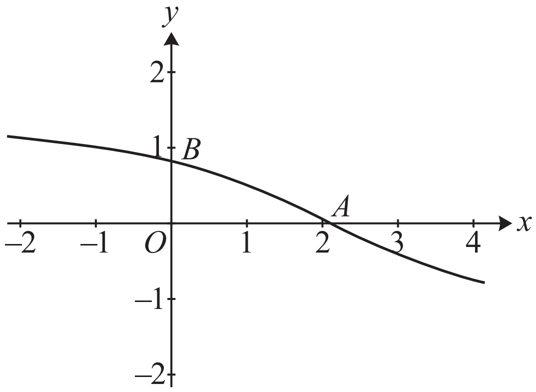
(c) Determine the range of values of x for which the expansion in part **(b)** is valid. **[2]**

9 A function f is defined for $x > 0$ by $\mathrm{f}(x) = \dfrac{6}{x^2+a}$, where a is a positive constant.

(a) Show that f is a decreasing function. **[4]**

(b) Find, in terms of a, the coordinates of the point of inflection on the curve $y = \mathrm{f}(x)$. **[5]**

10



The diagram shows the graph of $y = -\tan^{-1}\left(\frac{1}{2}x - \frac{1}{3}\pi\right)$, which crosses the x-axis at the point A and the y-axis at the point B.

(a) Determine the coordinates of the points A and B. **[3]**

(b) Give full details of a sequence of three geometrical transformations which transform the graph of $y = \tan^{-1}x$ to the graph of $y = -\tan^{-1}\left(\frac{1}{2}x - \frac{1}{3}\pi\right)$. **[3]**

The equation $x = -\tan^{-1}\left(\frac{1}{2}x - \frac{1}{3}\pi\right)$ has only one root.

(c) Show by calculation that this root lies between $x = 0$ and $x = 1$. **[2]**

(d) Use the iterative formula $x_{n+1} = -\tan^{-1}\left(\frac{1}{2}x_n - \frac{1}{3}\pi\right)$, with a suitable starting value, to find the root correct to 3 significant figures. Show the result of each iteration. **[3]**

(e) Using the diagram in the Printed Answer Booklet, show how the iterative process converges to the root. **[2]**

11 In this question you must show detailed reasoning.

A function f is given by $f(x) = \dfrac{x-4}{(x+2)(x-1)} + \dfrac{3x+1}{(x+3)(x-1)}$.

(a) Show that f(x) can be written as $\dfrac{2(2x+5)}{(x+2)(x+3)}$. **[5]**

(b) Given that $\int_a^{a+4} f(x)\,dx = 2\ln 3$, find the value of the positive constant a. **[7]**

12 (a) By first writing $\tan 3\theta$ as $\tan(2\theta+\theta)$, show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$. **[4]**

(b) Hence show that there are always exactly two different values of θ between 0° and 180° which satisfy the equation

$3\tan 3\theta = \tan\theta + k$,

where k is a non-zero constant. **[5]**

END OF QUESTION PAPER